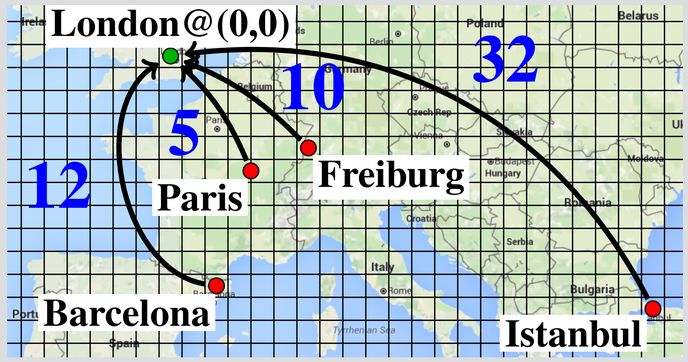
On the Compilability and Expressive Power of State-Dependent Action Costs

David Speck, David Borukhson, Robert Mattmüller und Bernhard Nebel

University of Freiburg, Germany – ⟨speckd, borukhd, mattmuel, nebel⟩@informatik.uni-freiburg.de

Motivation



- Planning with **state-dependent action costs** (SDAC)

Open Questions!

- How **computationally hard** is planning with SDAC?
- What is the **expressive power** of SDAC?
- Can SDAC simply be **compiled away**?

Planning with SDAC

An **SDAC planning task** is a 5-tuple $\langle \mathcal{P}, \mathcal{A}, \mathcal{C}, \mathcal{I}, \mathcal{G} \rangle$.

- $\mathcal{P}$: finite set of **propositional atoms**
- $\mathcal{A}$: finite set of **actions** $a = \langle \mathit{pre}(a), \mathit{eff}(a) \rangle \in \mathcal{A}$
- $\mathcal{C} : \mathcal{S} \times \mathcal{A} \mapsto \mathbb{N}_0$: **state-dependent** action cost function
- $\mathcal{I}$: **initial state** over $\mathcal{P}$
- $\mathcal{G} \subseteq \mathcal{P}$: **goal** description

SDAC Plan

- Applicable **sequence of actions** $\pi = \langle a_0, \ldots, a_n \rangle$
- Starting in $s_0 = \mathcal{I}$ and resulting in $s_n \in \mathcal{G}$
- **Plan cost** $c(\pi) = \sum_{i=0}^{n-1} \mathcal{C}(s_i, a_i)$

Complexity Results and Compilability

Planning with SDAC is **PSPACE-complete** if the **cost function** is in FPSPACE.

- **Hardness**: reduction of planning with unit costs
- **Membership**: nondeterministic Turing Machine
- **Same** complexity $\not\Rightarrow$ **same** expressive power
  - Metric: **conciseness** of **compilation**

Compilation Schemes

- **Translation** from one formalism to another
  - SDAC tasks $\rightsquigarrow$ **constant cost** tasks
- Preservation of
  - polynomial **task size**
  - **plan existence** and **plan cost**

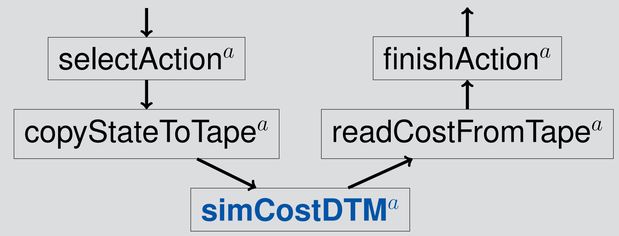
Is it **possible** to preserve the **plan length exactly**, **linearly**, or **polynomially**?

Compilability – Possibility Results

- Compile **each action** $a \in \mathcal{A}$ witch cost $\mathcal{C}(a, s)$
- **Simulate** a **DTM** computing $\mathcal{C}(a, s)$ within the task
- **Size** of new task is bounded by **space of the DTM**

- Cost function in **FP** $\rightsquigarrow$ **polynomial** plan length
- Cost function in **FPSPACE** $\rightsquigarrow$ **any** plan length

Compilation of Action a:



Impossibility Results

- SDAC tasks with **one action** a
- **Cost** of a **encodes instance** of
  - Parity (FP) or QBF (PSPACE)

With compilation **restrictions**:

- $\rightsquigarrow$ **Contradiction**
- $\rightsquigarrow$ **PH collapses** at **3rd level**

Overview

$\mathcal{C}$ complexity	Desired plan length preservation: linearly	polynomially	don't care
FP	**impossible**	**possible** using DTM compilation	
FPSPACE	**impossible** (unless PH collapses at the 3rd level)		**possible** using DTM compilation